



**Studies of a
Diffusive Model
for the
Dynamics of Stock Prices**

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RESearchers have been investigating a diffusive model with a price dependent diffusion coefficient to understand the occurrence of non-Gaussian price return distributions that are observed empirically in real financial markets. The work also applies to other physical processes that have non-Gaussian distributions of associated quantities, including temperature fluctuations in hard turbulence and diffusion in inhomogeneous media.

Empirical studies of price returns in financial markets have shown that their distribution is not Gaussian. Instead, they have either an exponential distribution, or a “fat-tailed” power-law distribution. In either case, large deviations occur more frequently than expected from a collection of independent, identically distributed finite events. To explain the occurrence of these empirical distributions, Levy flights have been proposed as the underlying dynamical processes responsible for the distributions of returns that are observed. Recently, however, an alternative dynamical model was proposed to explain the occurrence of the non-normal distributions. The model is a diffusive model with a price dependent diffusion coefficient. The continuum limit of the model can be solved exactly, and, depending on the functional form of the diffusion coefficient, we have shown that it can produce either exponential or power-law return distributions.

The model¹ is defined by considering a stock whose price at time t is given by $S(t)$. The “return” of the stock is then defined as $x(t) = \ln[S(t)/S_0]$, where S_0 is a reference price. The value of the return is assumed to have a diffusive dynamics in which the diffusion coefficient depends on the current value of the return. In particular, the model assumes that the diffusion coefficient can be expanded as

$$D(u) = D_0(1 + \varepsilon_1|u| + \varepsilon_2u^2 + \dots)$$

where $u = x/\sqrt{t}$ is a scaling variable, and the ε_i s are non-negative constants.

PHOTO ART—Dr. Kevin Bassler, assistant professor of Physics (UH), is a professional photographer. He is shown with photos hanging in his office showing the Giant Forest at the Sequoia National Park, the Canadian Rockies, and the Arches National Park in Utah. His photo of the Giant Forest was recently purchased for the permanent photo exhibit of the Museum of Fine Arts, Houston.

ISSO funds were used to support graduate student Angel L. Alejandro-Quinones for the summer months of 2004. Our work during that time led to two papers.^{2,3} In the first paper, submitted to *Physical Review E*, we show that continuum limit of the model can be solved exactly, and that, depending on the functional form of the diffusion coefficient, the price return distribution can be either an exponential, or a “fat-tailed” power-law. Furthermore, the range of critical exponents found that describe the “fat-tailed distributions” is exactly the range observed in real markets. In the second paper,³ which will also be orally presented in the upcoming SPIE conference “Fluctuations and Noise 2005” in Austin, Texas, in May, 2005, we investigate the effects of discreteness on the continuum limit solution. Real financial markets are, of course, discrete. Discrete distributions were determined using simulations as well as numerically exact calculations and compared to the corresponding distributions of the continuum model. Interestingly, a novel type of phase transition is discovered in discrete models that lead to “fat-tailed” distributions in the continuum limit, shedding light on the nature of such distributions. The transition is to a phase in which infinite price changes can occur in finite time.

References

- ¹J. L. McCauley and G. H. Gunaratne, “Theory of Fluctuations and Valuation in Their Options,” *Physica A* 329 (2003): 178.
- ²A. L. Alejandro-Quinones, K. E. Bassler, M. Field, J. L. McCauley, M. Nicol, I. Timofeyev, A. Torok, and G. Gunaratne, “A Theory of Fluctuations in Stock Prices,” *Physical Review E* (submitted).
- ³A. L. Alejandro-Quinones, K.E. Bassler, J. L. McCauley, and G. Gunaratne, “A Theory of Fluctuations in Stock Prices: Effects of Discreteness” *SPIE Proc., Fluctuations and Noise 2005* (publication scheduled).