A Neural Network-Based Approach for Control of Vibration in a Black Hawk Helicopter

Heidar Malki
Department of Engineering Technology

Karolos Grigoriadis
Department of Mechanical Engineering

José Canelon

Abstract—UH researchers devote themselves to the development of a time-domain approach for control of the vibration in a Black Hawk helicopter. Several schemes have been studied. The results show that this approach significantly reduces the vibration state after a short transient period after which the vibration is progressively reduced to almost zero.

Modern helicopters are essential transport vehicles because of their agility and maneuverability. However, helicopters have a significant rate of accidents and mishaps due to the severe level of vibrations of the main rotor, which produces large oscillatory forces and moments on the rotor blades, thereby causing excessive wear of these components and degraded ride quality, compromising the overall system reliability.

At present, most helicopters rely on passive methods which have reduced vibration but added significant weight penalty to the helicopter leaving the source of vibration uncontrolled.

Alternatively, some active control methods have been proposed, among which is the individual blade control (IBC),[1] in which the pitch angle of each blade is changed individually by an actuator. A control strategy is implemented based on a model of the vibration. Most of the reported applications use a linear model identified by least squares.

This approach has shown reasonably good performance at specific flight conditions, when a good linear model is identified. However, it is difficult to identify a single linear model for the full flight regime of the helicopter due to the nonlinear dynamics of the vibration. One of two schemes has been used to overcome this difficulty, based on the assumption that small changes in vibration can always be linearly related to small changes in the control inputs.

The first scheme is to schedule the linear model parameters according to the flight condition,[2] which requires measurement of the operating condition to determine which transfer matrix to use. This method is guaranteed to be stable and to work for all flight conditions identified. If there are only a few operating conditions, the approach is feasible; otherwise, it is quite cumbersome.

According to the second approach, a recursive identification method is employed to adaptively track changes in the transfer matrix, as flight conditions change.[3] As the vibration is reduced towards zero, the changes in control also approach zero, and the identification algorithms attempt to identify the matrix relating the small changes in control to the measurement noise, which is obviously the null matrix. This resolution produces erratic controller behavior.

Alternatively, it is also possible to model the nonlinear dynamics using nonlinear identification algorithms. A difficulty of this implementation is that the nonlinear model, once identified, must be linearized to be used in the control law. In this project, this difficulty is overcome by means of a neural network-based approach for time-domain control of the vibration throughout the full flight regime of a Black Hawk helicopter. In this approach, a neural network model of the helicopter vibration is first identified. Because of the universal approximation capabilities of neural networks,[4] one is able to identify a nonlinear model of the vibration for all operating conditions. Then, at every operating point of interest during the helicopter operation, an optimal local linear model is found by linearizing the neural network model. This optimal local linear model is used to design a local control input which will be applied to the actuators to reduce vibration. The scheme used to locally linearize the neural network model is based on the approach proposed by Teixeira and Zak,[5] which has been extended to neural networks models.

The same modeling and linearization approach was successfully used to build a model of the vibration dynamics using a frequency domain representation of the IBC input vector, i.e., (sine and cosine) coefficients are used to represent each harmonic in the input signal, rather than the time signal itself.[6] This representation may be cumbersome for control design purposes, since the dimension of the input vector is larger than in the time-domain representation.

A similar time-domain control approach is proposed with pole placement being used as the linear control design technique.[7] Although pole placement reduces the vibration to small final values after a short transient, such final values are not zero. This report mainly includes the results presented in Canelon and his colleagues[8] in which an optimal discrete linear quadratic tracker scheme[9] combined with an integrator and closed-loop poles placed in a desired circular region.[10] Results show that the proposed control approach is effective in significantly reducing the vibration state after a short transient after which, due to the action of the integrator, the vibration is progressively reduced to almost zero.

Methodology

Feedforward Neural Network Model of the Helicopter Vibration and Optimal Linear Model

First, a feedforward neural network discrete-time state-space model is identified relating the helicopter vibration to the individual blade control (IBC) inputs. Such model has the form,

$$\boldsymbol{x}(k+1) = \mathbf{F}(\boldsymbol{x}(k), \boldsymbol{u}(k)), \tag{1}$$

where $\mathbf{F}:\mathbf{R}^{n+m} \rightarrow \mathbf{R}^n$ represents the feedforward neural network mapping, and $\boldsymbol{x}(k) \in \mathbf{R}^n$ and $\boldsymbol{u}(k) \in \mathbf{R}^m$ are, respectively, the vibration state and the IBC input vector at time k.

Then, during the operation of the helicopter, the neural network model (1) is optimally linearized to yield, at each operating state of the helicopter, a local linear discrete-time state-space model of the form

$$\boldsymbol{x}(k+1) = \mathbf{A}_k\boldsymbol{x}(k) + \mathbf{B}_k\boldsymbol{u}(k) \tag{2}$$

where the subscript k indicates dependency of the model upon the particular vibration operating state. The optimal local linear models (1) have the exact dynamics of the nonlinear model (2) at the operating states of interest, with minimum modeling errors in their neighborhoods.

The optimal linearization approach used here is now described. For any operating state of the neural network model (1), it is important to find a local equivalent linear model of the form (2), where $\mathbf{A}_k$ and $\mathbf{B}_k$ are matrices of appropriate dimensions to be determined. At the operating point $(\boldsymbol{x}_k,\boldsymbol{u}_k)$, the following conditions must be satisfied:

$$\mathbf{F}(\boldsymbol{x}_k,\boldsymbol{u}_k) = \mathbf{A}_k\boldsymbol{x}(k) + \mathbf{B}_k\boldsymbol{u}(k). \tag{3}$$

In the neighborhood of the operating point, the following

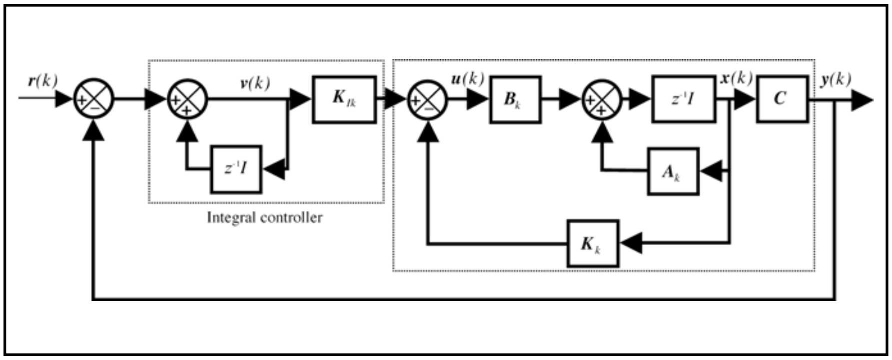


Figure 1. Optimal discrete linear quadratic tracker

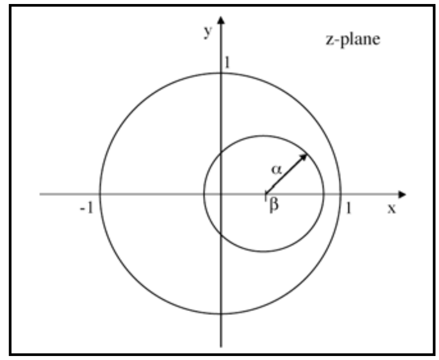


Figure 2. Circular region of radius α centered at β

conditions are satisfied:

$$\mathbf{F}(\boldsymbol{x},\boldsymbol{u}) \approx \mathbf{A}_k\boldsymbol{x} + \mathbf{B}_k\boldsymbol{u}, \quad (4)$$

where $(\boldsymbol{x},\boldsymbol{u})$ represents a point close to the operating point $(\boldsymbol{x}_k,\boldsymbol{u}_k)$. It can be shown that, according to the method of Lagrange multipliers, the jth rows of the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$, $\alpha^T{}_{jk}$ and $\beta^T{}_{jk}$, respectively, are given as

$$a^T_{jk} = \nabla_{x_k^T} f_j(x_k,u_k) + \frac{f_j(x_k,u_k) - \nabla_{x_k^T} f_j(x_k,u_k)x_k - \nabla_{u_k^T} f_j(x_k,u_k)u_k}{x_k^T x_k + u_k^T u_k} x_k^T \quad (5)$$

and

$$b^T_{jk} = \nabla_{u_k^T} f_j(x_k,u_k) + \frac{f_j(x_k,u_k) - \nabla_{x_k^T} f_j(x_k,u_k)x_k - \nabla_{u_k^T} f_j(x_k,u_k)u_k}{x_k^T x_k + u_k^T u_k} u_k^T. \quad (6)$$

Local Controller Design and
Overall Proposed Control Approach

The optimal linear model is used for the design of the controller to be applied to the helicopter at that particular operating state. Here we use a modified version of the optimal quadratic linear tracker,[9] depicted in Fig. 1, with the added feature of closed-loop poles placed inside a particular region. The control design procedure is now described.

Consider the discrete-time local linear model

$$\boldsymbol{x}(k+1) = \mathbf{A}_k\boldsymbol{x}(k) + \mathbf{B}_k\boldsymbol{u}(k) \quad (7a)$$
$$\boldsymbol{y}(k) = \mathbf{C}\boldsymbol{x}(k), \quad (7b)$$

where $\mathbf{C}$ is the identity matrix since the full vibration state is being controlled; i.e., the output $\boldsymbol{y}(k)$ is the vibration state.

The local linear control law to be applied to the helicopter at each particular operating state has the form:

$$\boldsymbol{u}(k) = -\boldsymbol{K}_k\boldsymbol{x}(k) + \boldsymbol{K}_{Ik}\boldsymbol{v}(k), \quad (8)$$

where $\boldsymbol{x}(k)$ is the vibration state at time k, $\boldsymbol{K}_k$ is the feedback gain, is the integral gain and, from Fig. 1,

$$\boldsymbol{v}(k) = -\boldsymbol{v}(k-1) + \boldsymbol{r}(k) - \boldsymbol{y}(k), \quad (9)$$

where $\boldsymbol{r}(k)$ is the set-point. Since the control task is to reduce the helicopter vibration as much as possible,

$$\boldsymbol{r}(k) = 0. \quad (10)$$

Hence, equation (9) reduces to

$$\boldsymbol{v}(k) = \boldsymbol{v}(k-1) - \boldsymbol{y}(k). \quad (11)$$

The design parameters $\boldsymbol{K}_k$ and $\boldsymbol{K}_{Ik}$ can be determined by solving the following Riccati equation for $\mathbf{P}_k$,
where $\mathbf{Q}$ is a positive definite or positive semidefinite matrix,

$$\hat{\mathbf{A}}_k^T\mathbf{P}_k\hat{\mathbf{A}}_k - \mathbf{P}_k + \mathbf{Q} - \hat{\mathbf{A}}_k^T\mathbf{P}_k\hat{\mathbf{B}}_k\left(\mathbf{R} + \hat{\mathbf{B}}_k^T\mathbf{P}_k\hat{\mathbf{B}}_k\right)^{-1}\hat{\mathbf{B}}_k^T\mathbf{P}_k\hat{\mathbf{A}}_k = 0 \quad , (12)$$

$\mathbf{R}$ is a positive definite matrix, and $\mathbf{A}_k$ and $\mathbf{B}_k$ are the matrices of the augmented system obtained by defining the augmented state

$$\hat{\mathbf{x}} = \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{v}(k) \end{bmatrix}.$$

These matrices are defined as

$$\hat{\mathbf{A}}_k = \begin{bmatrix} \boldsymbol{A}_k & 0 \\ -\boldsymbol{C}\boldsymbol{A}_k & \boldsymbol{I} \end{bmatrix}, \quad (13)$$

and

$$\hat{\mathbf{B}}_k = \begin{bmatrix} \boldsymbol{B}_k \\ -\boldsymbol{C}\boldsymbol{B}_k \end{bmatrix}. \quad (14)$$

The augmented control gain can be calculated as

$$\hat{\mathbf{K}}_k = [\mathbf{K}_k - \mathbf{K}_{Ik}] = \left(\mathbf{R} + \hat{\mathbf{B}}_k^T\mathbf{P}_k\hat{\mathbf{B}}_k\right)^{-1}\hat{\mathbf{B}}_k^T\mathbf{P}_k\hat{\mathbf{A}}_k. \quad (15)$$

According to the properties of optimal control design, the resulting closed-loop system is asymptotically stable; i.e., the closed-loop poles lie inside the unit circle. However, it is difficult to control the exact location of such poles, which, in particular cases, may lead to problems.

Lee and Lee[10] formulated a discrete optimal control approach which guarantees that the closed-loop eigenvalues lie inside a circular region centered at (β,0) with radius α, as shown in Fig. 2, by solving the optimal control problem described by equations (12) and (15) with matrices and being replaced by the modified matrices

$$\hat{\mathbf{A}}_{k\,mod} = \frac{1}{\alpha}\left[\hat{\mathbf{A}}_k - \beta \boldsymbol{I}\right] \tag{16}$$

and

$$\hat{\mathbf{B}}_{k\,mod} = \frac{1}{\alpha}\hat{\mathbf{B}}_k, \tag{17}$$

respectively.

The described design procedure is repeated at every state the helicopter goes through during its operation. The proposed control approach is summarized in Fig. 3.

Results and Discussion

Three vibration measurements are included in the vibration state: axial force (AF), side force (SF), and yaw moment (YM). In addition, there are four IBC inputs, one per helicopter blade. Hence, the neural network vibration model has seven inputs and three outputs.

The data were obtained during the wind tunnel testing of the Black Hawk (UH-60A) rotor in the NASA Ames Research Center 80- by 120-foot wind tunnel. Measurements are taken 256 times per rotor revolution. Two conditions were simulated: forward flight at 43 kts and descent flight at 75 kts. A total of 1000 training and 500 validation vectors were used to identify the model. A neural network with 50 hidden units was trained for 500 iterations.

Since the state is constituted by three vibration measurements, three integrators are being used, one per output. Hence, both $\mathbf{x}(k)$ and $\mathbf{v}(k)$ in (8) are three-dimensional vectors and the augmented state $\boldsymbol{x} \in \boldsymbol{R}^{6\times1}$.

Matrix $\boldsymbol{Q}$ had the form

$$\mathbf{Q} = \begin{bmatrix} q_x \boldsymbol{I}_3 & 0 \\ 0 & q_v \boldsymbol{I}_3 \end{bmatrix}, \tag{18}$$

while matrix $\boldsymbol{R}$ was of the form

$$\boldsymbol{R} = r\boldsymbol{I}_4, \tag{19}$$

where $\mathbf{I}_n$ represents the n-dimension identity matrix and q_x, q_v and r are constants that were adjusted by trial and error. We selected $q_x = 50\,000$, $q_r = 200$, and $r = 500$.

The parameters of the circular region were chosen as $\beta = 0.5$ and $\alpha = 1$. Although this circular region includes areas outside the unit circle, the optimal design procedure guarantees closed-loop poles inside the unit circle. In addition, it was observed that choosing a circular region of this size or bigger yielded integrator closed-loop poles very close to one, while the location of the remaining poles varied between 0.5 and 0.6.

Figures 4(a), 4(b) and 4(c) display the axial force, side

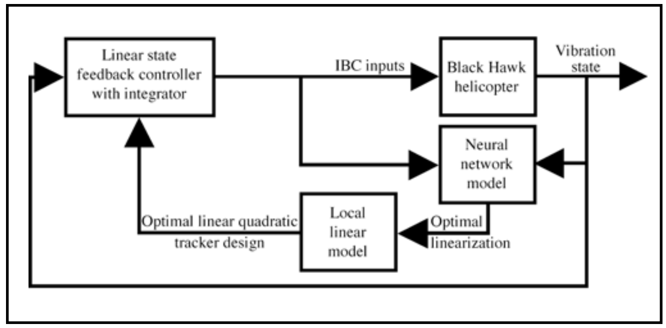


Figure 3. Proposed vibration control approach

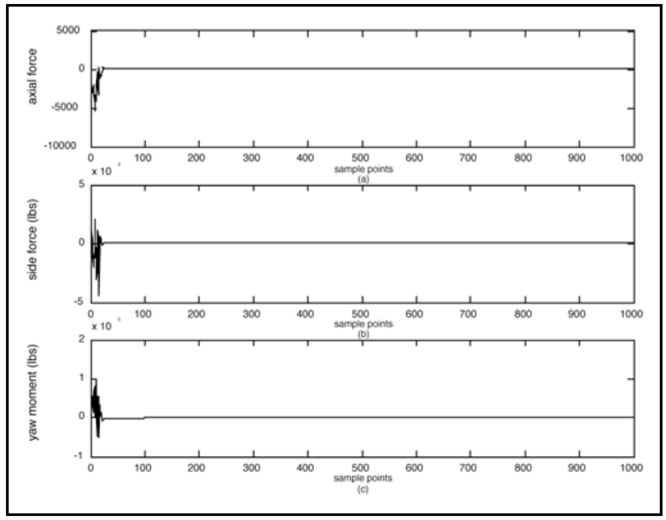


Figure 4. Vibration states during the control task using the optimal linear quadratic tracker: (a) axial force, (b) side force, and (c) yaw moment

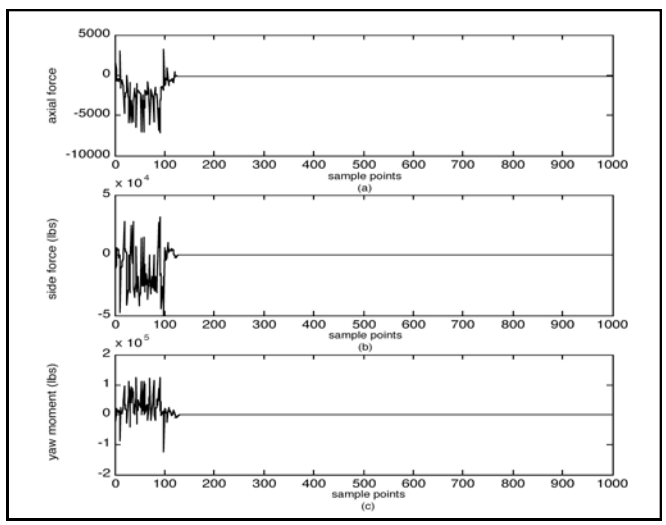


Figure 5. Vibration states during the control task using pole placement: (a) axial force, (b) side force, and (c) yaw moment

force, and yaw moment during the control task, when the optimal linear quadratic tracker is used as the control design technique. For comparison purposes, the results obtained with pole placement as the linear control design technique[7] are included in Figs. 5(a), 5(b) and 5(c). The desired pole locations are 0.9, 0.8, and 0.7.

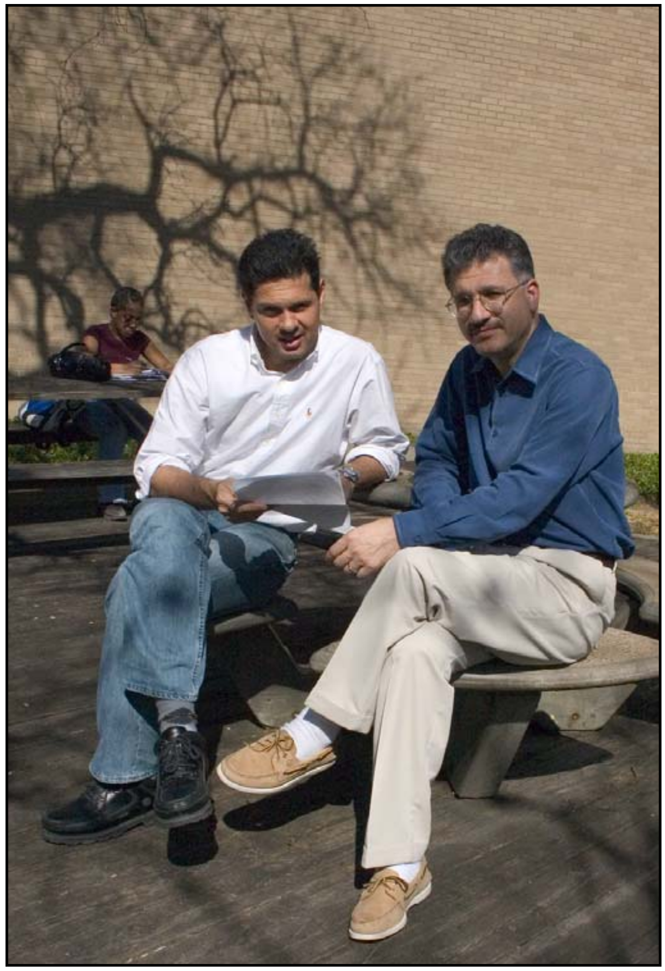
In both cases, the initial values of the vibration were chosen randomly within the interval of variations of the original data. Since the developed neural network model is the only available model of the helicopter vibration dynamics, it was used to test the proposed control approach.

It can be seen that during the control task using the optimal tracker, the vibration is reduced significantly after a transient of approximately 20 sample points, while for the pole placement case the reduction is achieved after a transient of 120 sample points. In addition, the final values of the vibration states for the optimal tracker are 0.17 lbs for the axial force, -4.5 lbs for the side force, and -1.8 lbs for the yaw moment, whose magnitude is smaller than the final values corresponding to the pole placement case, -125 lbs, 40 lbs, and 1250 lbs, respectively. Hence, the proposed control approach significantly reduces the vibration at both transient and steady states.

Conclusion

This report presented a neural network approach for the time-domain control of the vibration in a Black Hawk helicopter. Such an approach comprises the following steps: (1) identification of a neural network nonlinear model of the helicopter vibration dynamics, (2) optimal linearization of the neural network model at every operating point of the helicopter to yield a local linear model, and (3) design of the vibration control inputs based on such local linear model, by means of an optimal linear quadratic tracker scheme.

The results show that the proposed control approach is effective by significantly reducing the vibration state after a short transient, with final values very close to zero.



TECHNOLOGY—Karolos Grigoriadis, professor of mechanical engineering (*l.*), and Heidar Malki, professor of engineering technology (*r.*), have teamed up to solve problems of vibration in the Black Hawk helicopter. They are shown here in the arbor of the College of Technology.

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