PWM Control of Formation Flying Space Vehicle

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Abstract—The concept of Multiple Spacecraft Formation Flying (MSFF) has witnessed the development of a pulse-width-modulated control technique using optimal linear model-based digital redesign.

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Multiple spacecraft formation flying (MSFF) is emerging as a methodology for use in future space missions.[1] MSFF control methodologies are designed to conduct good tracking of relative position trajectories of a pair of spacecraft, a leader and follower.

Surveying the current state of MSFF control, the utilization of the PWM control for MSFF systems is not fully developed. The principle of equivalent area[2] may be adopted for pulse-width-modulating of an analog control signal. To virtually implement a pulse-width-modulator in this way, a control signal needs to be digitalized during a sampling period for eluding the non-causality between the principle of equivalent area and the pulse-width-modulator. However, as shown in Chang et al., it is clear that control performance is severely degraded when an analog control is digitalized by simply inserting an ideal sampler and a zero-order holder.[3] The proposed digital redesign (DR) technique can significantly improve the performance of PWM control systems.

To alleviate difficulties, we propose an alternative method—PWM control synthesis by using an optimal linearization model-based DR (OLM-based DR) technique. Simulation studies show the effectiveness of the proposed method.

Methodology

An Analog Nonlinear Control System and Its OLM

Nonlinear MSFF dynamics can be represented as follows:[1]

$$\begin{cases} \dot{x} = F(x) + G(x)u \\ y = Cx \end{cases}. \tag{1}$$

While Eqn. 1 admits nonlinearities for handling a wide class of dynamics, many useful analysis and synthesis methods rely on linear models. However, the Taylor expansion-based linearization approach usually results in an affine rather than linear model, even if the operating point is a system equilibrium. The following theorem can be used to optimally linearize Eqn. 1 on $x_{op} = x(kT) \in \mathbb{R}^n$, $k \in \mathbb{Z}_{>0}$.

Theorem 1 (**γ-stabilizable OLM**) *If there exist matrices $S_k = S_k^T > 0$, Z_k, X_k, and Y_k with appropriate dimensions, and a possibly small $\gamma_1 > 0$ such that the following minimization problem (MP) has solutions, (A_k, B_k) is OLM with guaranteed stabilizability on an operating point $x_{op} = x(kT)$, where $A_k = S_k^{-1}Z_k$, $B_k = G(x_{op})$, and $(\bullet)^T$ denotes the transposed element.*

MP 1: *Minimize* γ_1 subject to

$$\min_{S_k, Z_k, Y_k, X_k}$$

$$\begin{bmatrix} -\gamma_1 S_k & (\bullet)^T \\ \left(\nabla_{x_{op}^T} F(x_{op})\right)^T S_k - Z_k^T & -\gamma I \end{bmatrix} < 0$$

$$S_k F(x_{op}) - Z_k x_{op} = 0$$

$$Z_k^T + Z_k + Y_k^T B_k^T + B_k Y_k < 0$$

$$S_k B_k - B_k X_k = 0$$

LMI-Based Digital Redesign

Consider a linearized model given by Theorem 1 and an analog control:

$$\begin{cases} \dot{x}_c = A_k x_c + B_k u_c \\ y_c = C_k x_c \end{cases} \tag{2}$$

$$u_c = K_c x_c + E_c r. \tag{3}$$

By introducing an ideal sampler and a zero-order holder between a plant and a control, a desired digitally controlled hybrid system is represented by

$$\begin{cases} \dot{x}_d = A_k x_d + B_k u_d \\ y_d(kT) = C x_d(kT) \end{cases}, \tag{4}$$

where $u_d = u_d(kT)$ is a piecewise-constant control to be determined during the time interval $[kT, kT + T)$. In this study, the redesigned digital control takes the form

$$u_d = K_d^k x_d(kT) + E_d^k r(kT) \tag{5}$$

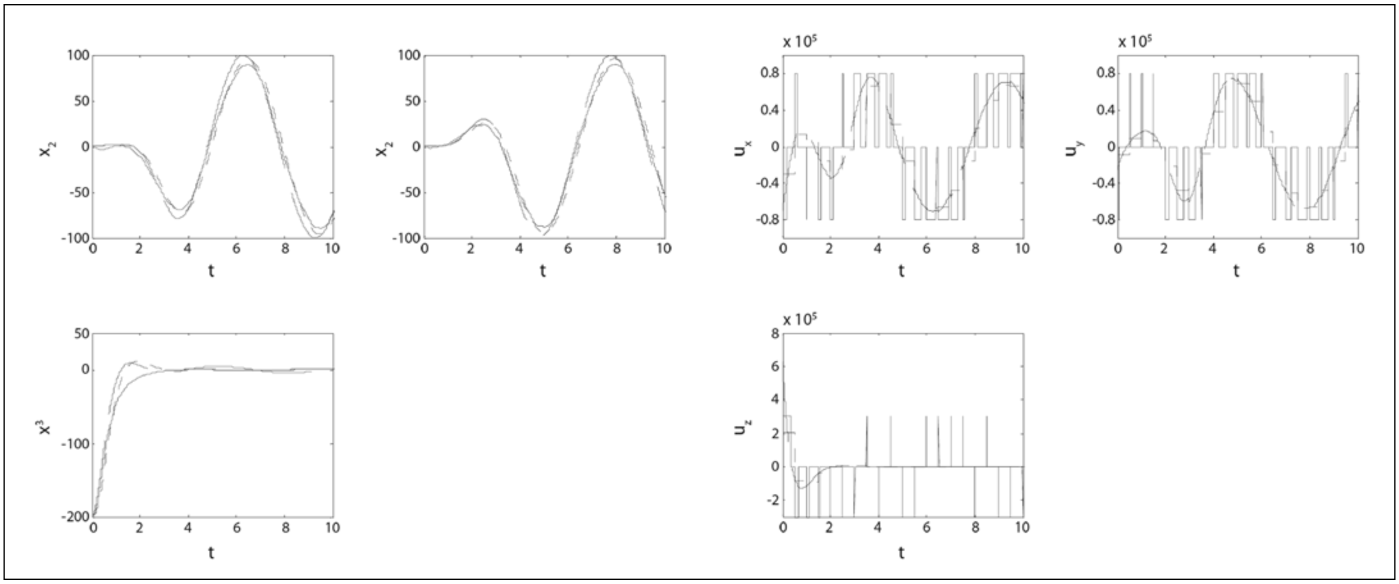

Figure 1. Time responses and controls by the proposed method for *T* = 0.5 h.

for the time interval $[kT, kT + T)$. Equivalent digital implementation of Eqn. 3 in the form of Eqn 5. can be done using the following DR:

Theorem 2 (**γ-Suboptimal DR for an OLM**) *If there exist matrices $P_k = P_k^T > 0$, K_d^k, and E_d^k with appropriate dimensions and, possibly, small scalars $\gamma_i > 0$, $i = 2,3$, such that the following MP has solutions, the state $x_d(kT)$ of Eqn. 4 by Eqn. 5 closely matches the state $x_c(kT)$ of the analogously controlled system (Eqn. 2) and Eqn. 3, and is exponentially stable.*

MP 2: *Minimize* $\gamma_2 + \gamma_3$ subject to

$$\min_{S_k, Z_k, Y_k, X_k}$$

$$\begin{bmatrix} -\gamma_2 I & (\bullet)^T \\ \phi_k - G_k - H_k K_d^k & -\gamma_2 I \end{bmatrix} < 0$$

$$\begin{bmatrix} -P_{k-1} I & (\bullet)^T \\ G_k + H_k K_d^k & -P_k^{-1} \end{bmatrix} < 0$$

$$\begin{bmatrix} -\gamma_3 I & (\bullet)^T \\ \psi_k - H_k E_d^k & -\gamma_3 I \end{bmatrix} < 0$$

PWM Control by DR: The Main Result

Now, we are interested in controlling the model

$$\begin{cases} \dot{x}_p = A_k x_p + B_k u_p \\ y_p = C x_p \end{cases} \tag{6}$$

by the PWM control mathematically represented by

$$u_p^j = \begin{cases} \operatorname{sgn}\left(u_d^j(kT)\right) M_j, & \text{for } t \in \left[kT, kT + T_k^j\right) \\ 0, & \text{otherwise} \end{cases}, \tag{7}$$

where u_p^j and u_d^j are the jth component of u_p and u_d, respectively.

One easy way to design an equivalent PWM control is to determine the firing duration time T_k^j by using the principle of equivalent area. However, there is the non-causal relationship between Eqn. 7 and the principle of equivalent area. Our strategy to resolve this problem is to use the DR: Find a digital control so that $x_c(kT)$ and $x_d(kT)$ are matched and identify T_k-decision logic so that $x_p(kT + T)$ in Eqn. 6 and Eqn. 7 is matched with $x_d(kT + T)$ in Eqn. 4 and Eqn. 5, with guaranteed stability.

Theorem 3 *If the following T_k-decision logic is equipped for* (7)

$$T_k^j = -T \frac{\left(1 - \dfrac{1 - \bar{\lambda}^{-1}(G_k)}{M_j} \left| u_d^j\left(x_p(kT)\right) \right| \right)}{\ln\left(\bar{\lambda}(G_k)\right)}$$

with a sufficiently small $T > 0$ satisfying

$$\sup_{k \in Z_{>0}} \left\| \left(e^{A_k T} \frac{\left(1 - \bar{\lambda}^{-1}(G_k)\right)}{\ln\left(\bar{\lambda}(G_k)\right)} B_k T - H_k \right) K_d^k \right\| < -\nu_2 + \sqrt{1 - e^{-2\zeta_2 T} + \nu_2^2}$$

the state $x_p(kT)$ of the PWM control system (Eqn. 6) with Eqn. 7 closely matches the state $x_c(kT)$ of the analogously controlled system (Eqn. 2) with Eqn. 3, and is exponential stable, where

$$\sup_{k \in Z_{>0}} \left\| G_k + H_k K_d^k \right\| \bar{\lambda}(\bullet)$$

is the absolute value of the largest eigenvalue.

Results and Discussion

We consider the dynamics of MSFF given by Queiroz et al. with the fractional placement errors of the on-off thruster nozzles.[1] The analog control gains for Eqn. 3 are designed by the standard linear quadratic regulation technique. A conventional direct digital implementation approach is also simulated for purposes of comparison.

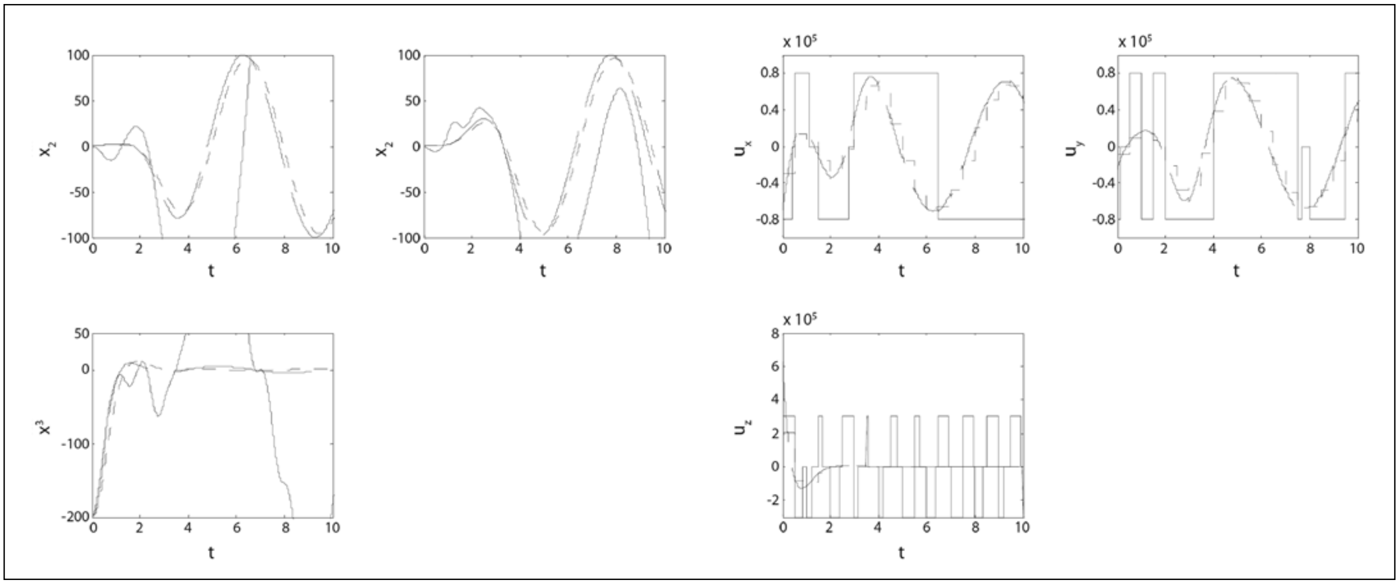

Figure 2. Time responses and controls by the simple digital implementation of analog control *T* = 0.5 h.

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The simulated time responses and applied controls by both methods for $T = 0.5$ are reported in Fig. 1. The trajectory of the follower relative to the leader, after the digitally redesigned PWM control is applied, perfectly tracks the desired reference orbit. In comparison with the direct digital implementation approach without utilizing any DR techniques, the simulated time responses and applied controls are shown in Fig. 2. Observe that tracking performances have been significantly degraded.

Conclusion

In this research, a new PWM control technology with the OLM-based DR technique has been presented for nonlinear MSFF. In the developed approach, the stabilizable OLM describing the local dynamic behavior of a nonlinear system is presented, which facilitates the application of the DR technique to nonlinear systems. A digital output-tracking control is obtained by using the newly developed LMI-based DR method. The redesigned digital control is converted to the equivalent PWM control so that the state controlled by the PWM control closely matches the state controlled by the pre-designed analog control. Simulation results have shown that with the proposed method an effective PWM control can be constructed for high performance systems that require a relatively long sampling period.

References

[1]M. S. Queiroz, V. Kapila, and Q. Yan, "Adaptive Nonlinear Control of Multiple Spacecraft Formation Flying," *J. Guidance, Control, and Dynamics* 23.3 (2000): 365-70.

[2]R. E. Andeen, "The Principle of Equivalent Areas," *Trans. on AIEE* 79 (1960): 332-36.

[3]W. Chang, J. B. Park, Y. H. Joo, and G. Chen, "Design of Sampled-Data Fuzzy-Model-Based Control Systems by Using Intelligent Digital Redesign," IEEE *Trans. on Circuits and Systems I* 49.4 (2002): 509-17.

[4]L. S. Shieh, I. C. Lin, and J. S. H. Tsai, "Design of PWM Controller for Sampled-Data System Using Digitally Redesigned PAM Controller," IEE *Proc*., Control Theory and Applications 142.6 (1995): 654-60.

Publications

Lee, H. J., R. S. Provence, L.-S. Shieh, and H. Malki. "Pulse-Width-Modulated Control of Nonlinear Multiple Spacecraft Formation Flying: Optimal Linearization Model-Based Digital Redesign Approach," *J. Guidance, Control, and Dynamics*. (*Submitted*.)

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Shieh, L. S. and H. A. Malki. "Adaptive Pulse-Width-Modulated Control of Multiple Spacecraft Formation Flying," NASA-JSC, 2007–2009, $150,000. (*To be submitted Oct. 2006*.)