

Computational Methods in Non-Smooth Mechanics: Applications to Dry Friction Constrained Motions

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Abstract—Non-smooth mechanical models involving dry friction constrained motion are important in various applications and are essential in the design of high-fidelity software simulation used on the Space Shuttle and the International Space Station. UHCL researchers introduce new friction models implemented with robust, efficient, and accurate simulation methods to address various issues, particularly with velocities near zero.

MOTIVATED BY THE NEED for real-time simulation of elasto-dynamical systems with friction, UHCL researchers consider the main goal of this project to mathematically analyze and numerically simulate the solution for non-smooth mechanical problems. Special attention is given to (1) the analysis of those differential equations and inequalities modeling¹⁻³ elasto-dynamical systems with dry friction and (2) the analysis and study of a family of numerical schemes enhanced with the existence of a friction multiplier. A family of numerical schemes is subsequently engaged in solving the existence of the new friction multiplier and providing for solutions.



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Methodology

Starting modestly, we initially modeled one degree-of-freedom for generalized systems.⁴ Since higher degree models give a better prediction of system behavior when velocities are near zero, our next step of study is to achieve a higher degree-of-freedom models. The methodology we successfully utilized in the study of one degree-of-freedom models is extended to the study of higher degree-of-freedom models by implementing more sophisticated friction models and computational techniques. Following our initial success on the one degree-of-freedom models, we have been continuously working on the higher degree-of-freedom models. In a previous article,¹ authors, including R. Glowinski and NASA engineers, investigated the use of multiplier techniques for a variety of dynamical systems with friction.

We have extended such research by considering a more

diverse nature of problems in elasto-dynamical systems. By incorporating inequalities modeling and dry-friction modeling, we can more precisely model the physical behavior of a system, particularly when the velocities are close to zero. We introduced and studied a family of numerical schemes enhanced with the existence of a friction multiplier. Indeed, the results obtained for two degrees-of-freedom models are very positive and were published^{1,2} in a refereed journal. In the study of higher degree-of-freedom models, we also observed that the rate of convergence in the computing multiplier tends to grow accordingly. Through further thorough analysis and study by introducing penalty terms on a convex domain, we incorporated the friction part with Newton methodology to acquire a fast rate of convergence for solving the multiplier. The mathematical analysis has proven this promising improvement, and the next step is to carry through the procedure numerically with test problems.

Results

Our motivation to investigate such problems is driven by two main factors: the applications of such problems and the computational methodology necessary to solve such problems. Presently, practitioners have to use existing in-house software, which is fundamentally inadequate, to model and implement their simulation process. Hence, the research is intended to produce important results that are of great interest to various government agencies, especially NASA. We have published preliminary results of one-degree and two-degree of freedom generalized test systems proposed by NASA engineers with promising results.^{1,2}

We have continued developing and studying more sophisticated models with more efficient numerical schemes. The results of test problems in two degree of freedom are also very encouraging. The development and analysis of higher number degrees-of-freedom models, typically allowing 10 to 20 degrees-of-freedom, and a subsequent evolution to beam-based flexible sys-

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tems—some ODEs become PDEs—will undoubtedly be of more significance and benefit to NASA's needs and practices. Therefore, it is essential to ensure that the rate of convergence on the multiplier is efficient. It will be the main focus of this proposed study. In the future, we will investigate theoretically the extension of the method in the first step to the simulation of visco-plastic particulate flow encountered in oil drilling technologies.

The computer implementation of methods resulting from these investigations will be part of another project. Among consideration in the difficulties of these problems is the solution of 3-dimensional non-smooth generalizations of the Navier-Stokes equations.

References

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Funding and Proposals

"Computational Methods in Non-Smooth Mechanics with Dry Friction Constrained Motions," Texas Higher Education Coordinating Board (under ARP), \$31,700.