Computational Methods in Non-Smooth Mechanics: Applications to Dry Friction Constrained Motions

by LieJune Shiau

ABSTRACT— NASA space science researchers need more sophisticated friction models and computational techniques. New models offer a better description of system behavior when velocities are close to zero. With improved numerical computational techniques, science can better solve higher-dimensional problems.

LieJune Shiau

Motivated by the need for real-time simulation of elasto-dynamical systems with friction, researchers seek to mathematically analyze and numerically simulate the solution of non-smooth mechanical problems. Special attention is given to those differential equations and inequalities modeling[1,2,3] elasto-dynamical systems with dry-friction. Thus, a family of numerical schemes with the existence of a friction multiplier is currently being analyzed and studied. This family of numerical schemes is subsequently engaged in solving the existence of the new friction multiplier as well as the solutions. Furthermore, with improved numerical computational techniques, higher dimensional problems can be simulated and resolved more efficiently.

Methodology

We formulate the following friction constrained motion model to describe some remote manipulator system simulators with finite number of degree of freedom:

$$\begin{cases} M\ddot{X} + AX + C(\operatorname{sgn}(\dot{X}) - \gamma(\dot{X})) = f \quad \text{on } (0,T), \\ X(0) = X_0, \dot{X}(0) = V_0, \end{cases}$$

where:

(1) X is a *displacement* (here $X(t) \in R^d$),

(2) the *mass matrix* M is symmetric and positive definite,

(3) the *stiffness matrix* A is symmetric and positive semi-definite,

(4) the *friction matrix* C is diagonal, i.e. $C = diag(c_1 \cdots, c_d)$, with $c_i \geq 0, \forall i = 1, \cdots, d$ and $\sum_{i=1}^{d} c_i > 0$,

(5) $\operatorname{sgn}(V) = \{\operatorname{sgn}(v_i)\}_{i=1}^{d}, \ \forall \ V = \{v_i\}_{i=1}^{d} \in R^d$,

(6) $\gamma(V) = \{\gamma_i(v_i)\}_{i=1}^{d} \ \forall \ V = \{v_i\}_{i=1}^{d} \in R^d, \gamma_i$ being a *non-decreasing Lipschitz* continuous function vanishing at 0 and such that $\lim_{\xi \to \pm\infty} \gamma_i(\xi) = \pm\beta_i$, with $0 < \beta_i < 1$,

(7) f is an *external force* such that $f \in L^2_{loc}(0,T;R^d)$, $T \in (0,+\infty)$,

(8) X_0 and V_0 belong both to R^d.

Starting modestly, we initially modeled one-degree and two-degree-of-freedom generalized systems.[4,5] Since higher degree models give a better prediction of the system behavior when velocities are near zero, we are then ready to move our focus to higher degree of freedom models. The methodology we successfully utilized in the study of one- or two-degree-of-freedom models is extended to the study of higher degree-of-freedom models by implementing more sophisticated friction models and computational techniques. In a higher degree-of-freedom case, the computational efficiency becomes an important issue. We then incorporate the penalty/Newton methodology developed in Dacorogna, et al. and Glowinski and his University of Houston colleagues to resolve this obstacle.

We illustrate an example of the 3-D test problem, as follows:

$$\begin{cases} M\ddot{X} + AX + C(\lambda - \gamma(\dot{X})) = F \\ |\lambda_i| \leq 1, \ \forall i = 1,2,3, \ \lambda \cdot \dot{X} = |X|, \\ X(0) - X_0, \dot{X}(0) = V_0, \end{cases}$$

with

- $M = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}, A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}, C = I$
- $\gamma \neq 0$, as previously defined in (3)
- $f = \{f_i\}_{i=1}^{3}$, where

(Equation continues on following page)

$$f_1(t)=\begin{cases}-1+2t=t^2-\gamma(1-t) & \text{if } 0\le t\le 1,\\ 1+(t-\frac{3}{2})-\gamma(0) & \text{if } 1\le t\le 2,\\ -78+70t-23t^2+3t^3-\gamma(4(t-3)(t-2)) & \text{if } 2\le t\le 3,\\ -1-\frac{1}{2}-3t^2+6t+\frac{t^3}{3}-\gamma(0) & \text{if } 3\le t\le 4,\end{cases}$$

$$f_2(t)=\begin{cases}-t+\frac{t^2}{2}-\gamma(0) & \text{if } 0\le t\le 1,\\ 2t-2t-\frac{t^2}{2}-\gamma(0) & \text{if } 1\le t\le 2,\\ -2t^3+16t^2-36t+26-\gamma(1-(t-3)^2) & \text{if } 2\le t\le 3,\\ 10-5t+2t^2-\frac{t3}{3}-\gamma(1-(t-3)^2) & \text{if } 3\le t\le 4,\end{cases}$$

$$f_3(t)=\begin{cases}-1-2t-\gamma(t-2) & \text{if } 0\le t\le 1,\\ 1-4t+t^2-\gamma(t-2) & \text{if } 1\le t\le 2,\\ \frac{-14+18t-9t^2+t^2}{3}-\gamma(0) & \text{if } 2\le t\le 3,\\ \frac{106}{3}-24t+5t^2-\frac{t^3}{3}-\gamma(1-(t-4)^2) & \text{if } 3\le t\le 4,\end{cases}$$

- $X_0=\{0,0,0\},\ V_0=\{1,0,-1\}.$

Our schemes are applied to this test problem including the improved penalty/Newton method studied. No more than four Newton's iterations were required at each time-step, making this approach much faster than the previous algorithms without the improvement. Numerical results are shown in Figs. 1-3.

Results

Our motivation to investigate such problems is driven by two main factors: the applications of such problems and the computational methodology necessary to solve such problems. Presently, practitioners are limited to the use of existing in-house software, which is fundamentally inadequate, to model and implement their simulation process. Hence, the proposed research will produce important results of interest to various government agencies, especially NASA. We have published results of a low degree of freedom. The current study resulted in the new publication of higher degree of freedom generalized test systems proposed by NASA engineers. These results are very promising.[4,5,6,7]

The development and analysis of higher number degrees of freedom models, typically allowing 10 to 20 degrees of freedom, and a subsequent evolution to beam-based flexible systems (some ODEs become PDEs) will undoubtedly be of more significance and benefit to NASA's needs and practices. Therefore it is essential to ensure that the rate of convergence on the multiplier is efficient; it is the main focus of the future study.

In the future, we will also investigate theoretically the extension of the method in the first step to the simulation of visco-plastic particulate flow encountered in oil drilling technologies. The computer implementation of the methods resulting from these investigations will be part of another project. Among consideration in the difficulty of these problems is the solution of 3-dimensional non-smooth generalizations of the Navier-Stokes equations.

References

[1]E. J. Dean, R. Glowinski, Y. M. Kuo, and G. Nasser, "Multiplier Techniques for Some Dynamical System with Dry Friction," *C. R. Acad. Sc.*, Paris, T. 314, *Série* I (1992): 153-59.

[2]E. J. Dean, R. Glowinski, Y. M. Kuo, and G. Nasser, "On the Discretization of Some Second Order in Time Differential Equations—Applications to Nonlinear Wave Problems, in Computational Techniques," in *Identification and Control of Flexible Flight Structures*. Ed. A.V. Balakrisknan. Los Angeles: Optimization Software, Inc., 1990. 199-246.

[3]G. Duvaut, J.-L. Lions, *Inequalities in Mechanics and Physics*. Berlin: Springer-Verlag, 1976.

[4]R. Glowinski, L. J. Shiau, Y .M. Kuo, and G. Nasser, "The Numerical Simulation of Friction Constrained Motions (I): One Degree of Freedom Models," *Appl. Mathematics Lett*. 17.7 (2004): 801-07.

[5]R. Glowinski, L. J. Shiau, Y. M. Kuo, and G. Nasser, "On the Numerical Simulation of Friction Constrained Motions(II): Multiple Degrees of Freedom Models," *Appl. Mathematics Lett*. 18.10 (2005): 1108-15.

[6]L. J. Shiau and R. Glowinski, "Operator Splitting Method for Friction Constrained Dynamical Systems," AIMS *Proceedings* (2005) 806-15.

[7]R. Glowinski, L. J. Shiau, Y. M. Kuo, and G. Nasser, "On the Numerical Simulation of Friction Constrained Motions," *Nonlinearity* 19 (2006): 195-216.

[8]B. Dacorogna, R. Glowinski, Y. Kuznetsov, and T .W. Pan, "On a Conjugate Gradient/Newton/Penalty Method for the Solution of Obstacle Problem: Application to the Solution of an Eikonal System with Dirichlet Boundary Conditions," in *Conjugate Gradient Algorithms and Finite Element Methods*. Ed. M. Kvrivzek, P. Neittaanmaki, R. Glowinski, and S. Korotov. Berlin: Springer, (2004): 263-83.

[9]R. Glowinski, Y. Kuznetsov, and T. W. Pan, "On a Penalty/Newton/Conjugate Gradient Method for the Solution of Obstacle Problems," *C.R. Acad. Sci.* Paris, *Série* I 336.5 (2003): 435-40.

Publications

Glowinski, R., L.J. Shiau, Y.M. Kuo, G. Nasser, "On the Numerical Simulation of Friction Constrained Motions," *Nonlinearity* 19 (2006): 195-216.

Presentations

Glowinski, R., L. J. Shiau, "Operator Splitting Method for Friction Constrained Dynamical Systems," AIMS Conference, Poitier, France, June 2006.

Proposals

Shiau, L. J. (P.I.) and R. Glowinski (CO-PI), "Computational Methods in Non-Smooth Mechanics: Application to Dry Friction Constrained Motions," ARP 2005. (*Unfunded.*)

SHELVING—Scientific periodicals in the University library.

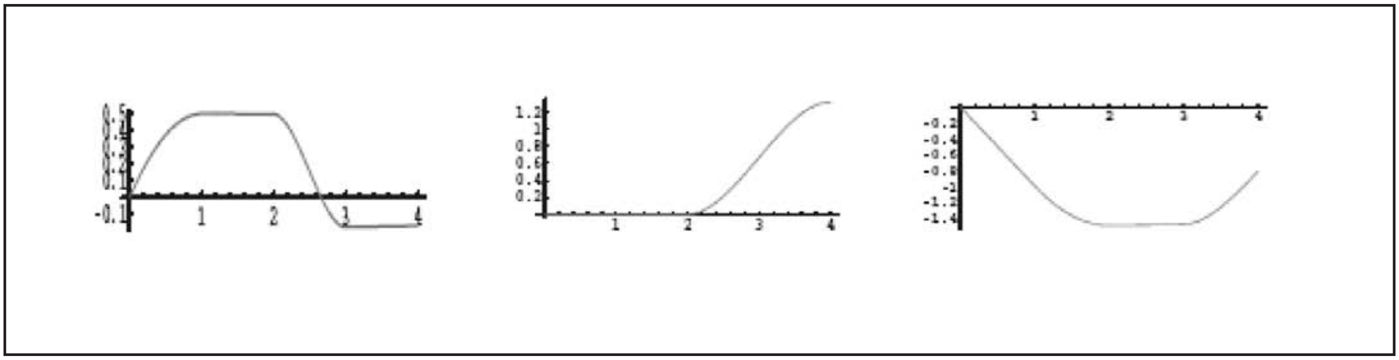

Figure 1.Left, the computed $x_1(t)$; middle, the computed $x_2(t)$; right, the computed $x_3(t)$

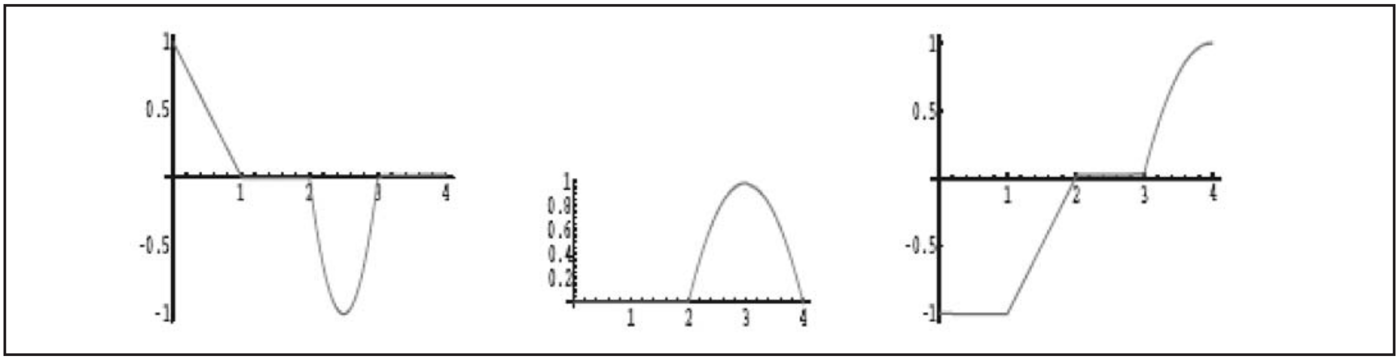

Figure 2.Left, the computed $v_1(t)$; middle, the computed $v_2(t)$; right, the computed $v_3(t)$

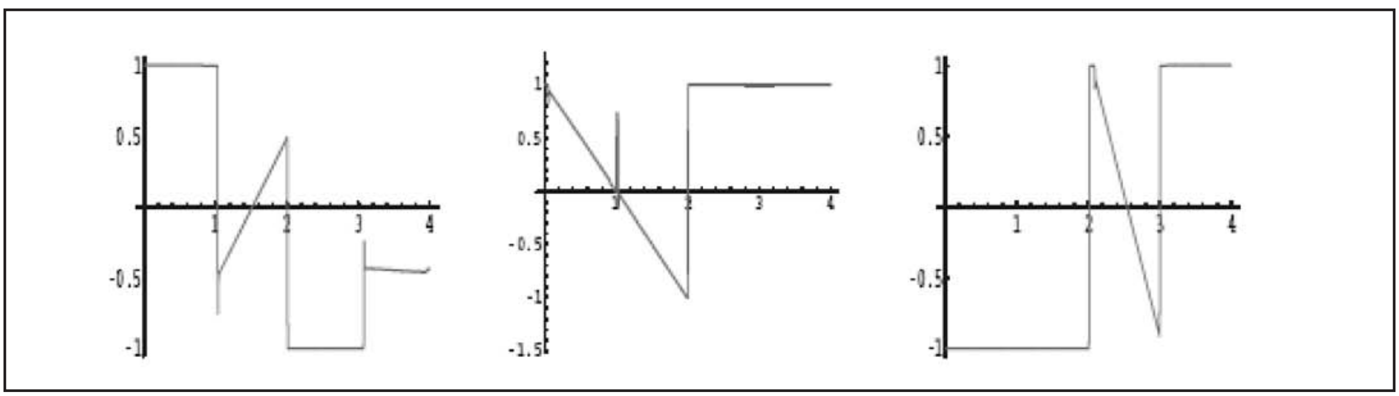

Figure 3. (*Left*) the computed $\lambda_1(t)$; (*middle*) the computed $\lambda_2(t)$; (*right*) the computed $\lambda_3(t)$.