

# Fault-Tolerant Control of a Truss Structure Using MR Dampers

by Gangbing Song and Linsheng Huo

**T**RUSS-TYPE STRUCTURES ARE OFTEN USED IN space applications. These structures can support interferometer, antenna, and other vibration-sensitive instrumentation. Launch constraints mandate these truss structures be lightweight. The combination of large and lightweight design results in these space structures being flexible and having low-frequency fundamental modes. These modes might be excited in a variety of tasks such as slewing, pointing maneuvers, and docking with other spacecraft. The induced vibration must be effectively suppressed to satisfy stringent requirements for attitude control and vibration sensitive missions, such as space-based interferometer. Various active vibration suppression strategies have been proposed in the past to suppress vibration of the truss structures. One semi-active device that appears to be particularly promising for truss applications is the Magneto-Rheological (MR) damper.

MR dampers have been developed in recent years as semi-active vibration devices.<sup>1,2,3</sup> For their operating fluid, they use a Magneto-Rheological fluid, a material which responds to applied magnetic fields. MR fluids alter their viscosity according to the applied magnetic field and exhibit nonlinear properties like a typical Bingham fluid. In MR dampers, electromagnets are used to generate the required magnetic field. The force generated in the MR damper is therefore controlled by adjusting the electric current supplied to the electromagnets.<sup>4</sup> The advantage of this adaptive-passive system lies in its fail-safe design. Unlike the active control, which requires active energy for vibration suppression, this MR fluid design will utilize its passive damping properties in the event of energy loss.

Fault tolerant schemes in engineering systems provide early warnings of faulty sensors, actuators, or system components. In this research, a “component fault” refers to a change in the operating behavior of a component such that the new behavior differs significantly from what is defined as normal behavior for that component. Common examples of such faults include bias errors in the output of a sensing device and loss of function for an actuating device.

Health monitoring systems are needed to provide early alarm notification of faults before they lead to catastrophic failure, so that remedial actions can be carried out to retain system stability



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**ABSTRACT**—In recent years, Magneto-Rheological (MR) dampers have been used for structural control. This research presents a new approach to vibration control of a space truss structure using MR dampers with system faults. The new approach uses an  $H_\infty$ -based Fault Detection and Isolation (FDI) algorithm to identify the faults and an  $H_\infty$ -based Fault Tolerant Controller (FTC) to achieve satisfactory vibration suppression in the presence of the faults. Simulation results of the proposed FDI algorithm and FTC controller shows its effectiveness for vibration suppression of a faulty truss for the faulty system.

and performance. Consequently, fault detection and isolation (FDI) and fault tolerant control (FTC) problems have received considerable attention in the control systems literature.<sup>5</sup>

Recently developed analytical redundancy techniques use the residue signals to monitor the health of systems. The term *residue* is defined as a signal that is zero when the system functions properly and nonzero when some abnormal behavior is observed. This residual signal indicates the faulty information in the system; it can be used not only for fault detection, but also for fault identification. Furthermore, it provides basic information for the fault tolerant control purpose of use. It is a natural way to cope with the FTC problem thereby employing the fault diagnosis information online.

There are several approaches to generating the residual signals and building the FTC scheme, such as the unknown input observer, the  $H_\infty$  method, the Kalman filter, and the  $H_2/LQG$  techniques.<sup>6</sup> Among these approaches,  $H_\infty$  optimization-based

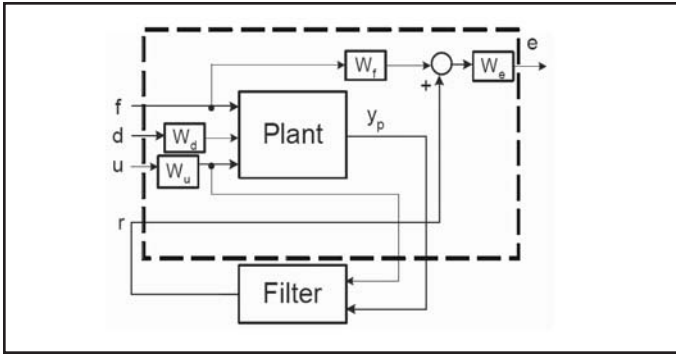


Figure 1. Block Diagram of the  $H_\infty$  Fault Detection Filter

methods have attracted much attention by their explicit address of robustness issues. In this work, an  $H_\infty$  FDI filter is developed based on the system identification model of the truss. The main design goal of this FDI filter is to detect and identify sensors failure in the truss structure system. The linear matrix inequality formulation of the FDI  $H_\infty$  filter is obtained based on the famous bounded real lemma.<sup>7</sup> Researchers propose an FTC  $H_\infty$  control scheme that successfully retains the vibration suppression level when the sensors on the truss partially fails.

### Methodology

Consider the following state-space realization for a plant  $P$  given by

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u + E_p d + F_p f \\ y_p &= C_p x_p + D_p u + G_p d + H_p f \end{aligned} \quad (1)$$

where  $x_p$  is the state vector,  $u$  is the control input,  $d$  is the disturbance input,  $f$  is the fault input,  $y_p$  is the system output, and  $A_p$ ,  $B_p$ ,  $E_p$ ,  $F_p$ ,  $C_p$ ,  $D_p$ ,  $G_p$  and  $H_p$  are real matrices of appropriate dimensions. Suppose  $F$  is the unknown filter to be determined and has the following state-space representation,

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_f u + E_f y \\ r &= C_f x_f + D_f u + H_f y \end{aligned} \quad (2)$$

where  $x_f$  is the filter state vector,  $u$  is the control input,  $r$  is the output vector and  $A_f$ ,  $B_f$ ,  $E_f$ ,  $C_f$ ,  $D_f$ , and  $H_f$  are real matrices of appropriate dimensions. Define the generalized disturbance,  $\omega^T = [u, d, f]^T$ , and estimation error,  $e = r - f$ , as shown in Fig. 1. The problem with the  $H_\infty$  optimal filter is to find a dynamic filter to minimize the worst case estimation error energy over the energy of bounded generalized disturbance. That is,

$$\min_F \sup_{\omega \in L_2} \frac{\|e\|_2}{\|\omega\|_2} \quad (3)$$

The expression (3) is equivalent to minimizing the  $H_\infty$  norm of the transfer function  $T_{\omega e}$  between the generalized disturbance

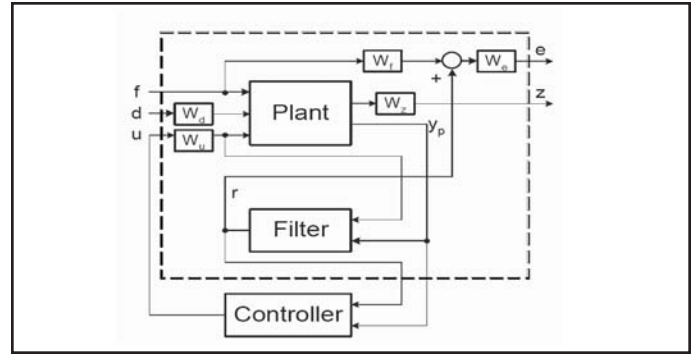


Figure 2. Block Diagram of the Proposed FTC.<sup>8</sup>

input and the error of the fault estimation. Similarly, the problem raised in  $\gamma$ -suboptimal  $H_\infty$  FDI filtering is to find, if there exists, a filter,  $F$ , such that

$$\|T_{\omega e}\| < \gamma \quad ,$$

where  $\gamma$  is a given positive scalar.

Using the linear fractional transformation (LFT), the space state realization of the transfer function (or transfer matrix)  $T_{\omega e}$  (see the black square in Fig. 1) can be obtained as

$$\begin{aligned} \dot{x}_s &= A_p x_s + B_\omega \omega \\ e &= D_{e\omega} \omega + r \\ y &= C_y x_s + D_{y\omega} \omega \end{aligned} \quad (4)$$

where

$$x_s = x_p, \quad B_\omega = [B_p, E_p, F_p], \quad D_{e\omega} = [0, 0, -I],$$

$$C_y = [0, C_p]^T \quad \text{and}$$

$$D_{y\omega} = \begin{bmatrix} I & 0 & 0 \\ D_p & G_p & H_p \end{bmatrix}, \quad \text{i.e. } y^T = [u, y_p]^T.$$

Thus, the optimal or  $\gamma$ -suboptimal  $H_\infty$  FDI filtering problem can be formulated as the standard  $H_\infty$  control problem and then solved by the linear matrix inequality (LMI) approach.

It is well known that the solution to the  $H_\infty$  control problem is a powerful tool in solving the disturbance attenuation problem. The  $H_\infty$  optimal fault tolerant control design requires a dynamic controller,  $C$ , to minimize the worst case performance output energy over the energy of bounded generalized disturbance,  $\omega_c^T = [f, d]^T$ . That is,

$$\min_C \sup_{\omega_c \in L_2} \frac{\|z\|_2}{\|\omega_c\|_2} \quad (5)$$

The control strategy of the  $H_\infty$  control FTC controller is shown in Fig.2.

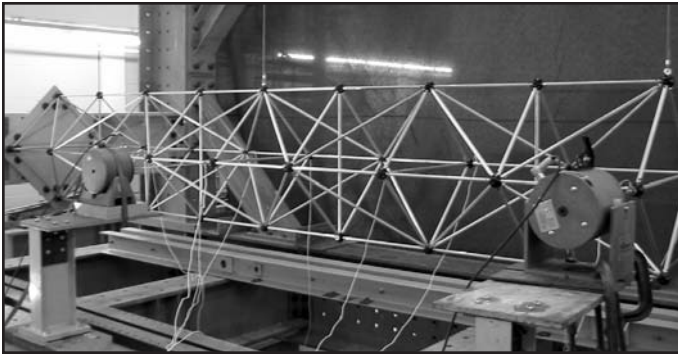


Figure 3. 8-Bay Truss at Rice University

### Fault-Tolerant Control of a Truss Structure

The truss used in this research is located at Rice University as shown in Fig. 3. This 8-bay, planar aluminum truss structure consists of 109 rod elements connected at 36 nodes. The total length of the truss is 4m with each bay 0.5m long, and with rod elements having a Young's modulus of  $E=7.58 \times 10^7 \text{ N/m}^2$ . All the members are 0.1 in. thick hollow tubes with an outer diameter of 0.5 in.

The truss can be simplified as a finite element model with 36 nodes and 109 members as shown in Fig. 4. This truss has an electromechanical shaker mounted for excitation at node 4. Four accelerometers are mounted for monitoring of vibrations on the truss at node 1, 9, 13 and 17. A Magneto-Rheological (MR) damper is installed between node 32 and 36 to reduce the vibration of the truss.

Determining the transfer function using the mathematical modeling and finite element analysis is complex. The finite element models sometimes are not feasible for control purposes because their orders are too high to achieve the desired accuracy. However, the system identification technique based on experimentation offers a rather simplistic approach to obtain the transfer function of the system. Input and output signals from the system are analyzed in order to obtain a model.

In this work, the system identification algorithm used to identify the system is based on the Subspace method. A linear system can be represented in the state space innovations form as

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ke(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \quad (6)$$

*i.e.*, where  $e(t)$  is the innovation (*i.e.*, the part of the output that cannot be predicted from the past data),  $x(t)$  is the state vector,  $y(t)$  is output,  $u(t)$  is the input and  $K$  is the Kalman gain.

The subspace method can be used to estimate the  $A$ ,  $B$ ,  $C$ ,  $D$  and  $K$  matrices. Assuming that  $x(t)$ ,  $y(t)$ , and  $u(t)$  are known, equation (6) becomes a linear regression. This will enable us to estimate the matrices  $C$  and  $D$  by the least square method and will lead us to determine  $e(t)$ . Again,  $e(t)$  can be treated as a known signal, and this will lead to the determination of  $A$ ,  $B$  and  $K$  using the least squares method. The Kalman gain  $K$  is computed using the Riccati equation. In the above method, initially it is assumed that states  $x(t)$  are known, but they need to be determined. The states  $x(t)$  can be

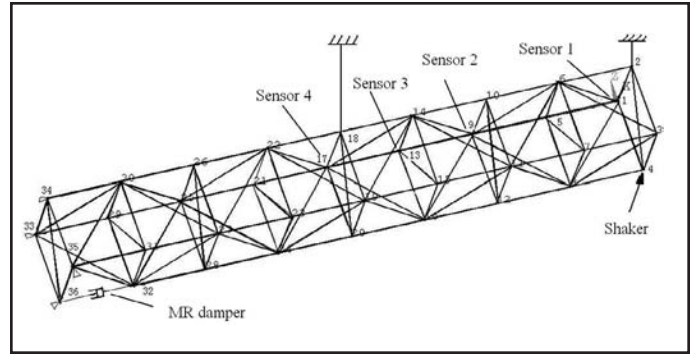


Figure 4. Finite Element Model of the Truss

formed as linear combinations of the  $k$  step ahead of predicted outputs. The predictor, in this method, can be determined using the  $k$  step ahead of predictors by projections from the observed data sequences. The above model derived from the subspace method is then used as a base model for further refining the model by the prediction error method (PEM). In the time domain, the above system can be represented by using the shift operator  $q$  as

$$\begin{aligned} y(t) &= G(q)u(t) + H(q)e(t) \\ G(q) &= C(qI - A)^{-1}B + D \\ H(q) &= C(qI - A)^{-1}K + I \end{aligned} \quad (7)$$

where  $G(q)$  is the transfer function of the system,  $e(t)$  is the innovation, and  $I$  is the identity matrix. From the observed data of input,  $u$ , and output,  $y$ , the prediction errors can be computed as

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)] \quad (8)$$

The above error can now be parameterized by the state space matrices derived by the subspace method. The common parametric identification method is to determine estimates of  $G$  and  $H$  by minimizing

$$V_N(G, H) = \sum_{t=1}^N e^2(t) \quad (9)$$

This algorithm forms the basis for the prediction error method. The model is first initialized and further adjusted by optimizing the prediction error fit. Substantial details for system identification can be found in the MATLAB reference manual. MATLAB has the System Identification toolbox to perform the above algorithm. The prediction error method first initializes the model by using the subspace algorithm and then minimizes the prediction error. In this research, a state space realization that does not model the noise properties (*i.e.*, an output error model,  $K = 0$ ), is considered. Thus, the implications will be derived from the predictors which will be based only on the past inputs.

Using the truss structure discussed above, the shaker is excited at node 4 by a sweep sine signal of frequency ranging

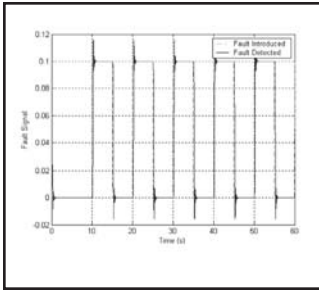


Figure 5. Fault ID in Sensor 1

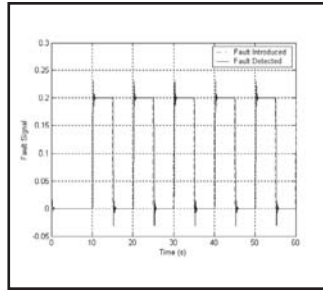


Figure 6. Fault ID in Sensor 2

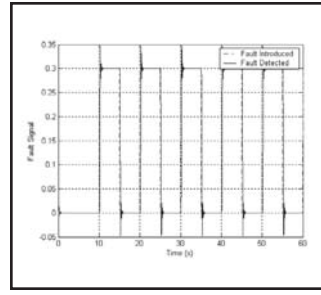


Figure 7. Fault ID in Sensor 3

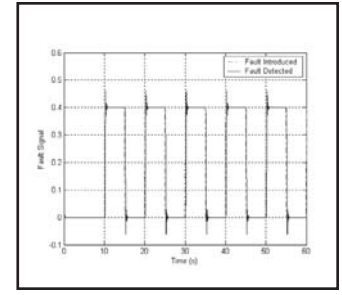


Figure 8. Fault ID in Sensor 4

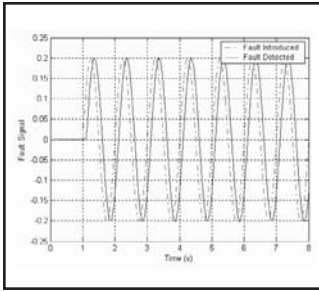


Figure 9. Fault ID in Sensor 1

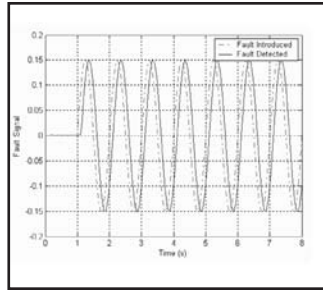


Figure 10. Fault ID in Sensor 2

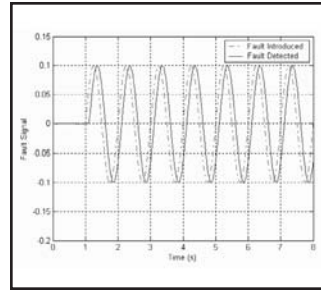


Figure 11. Fault ID in Sensor 3

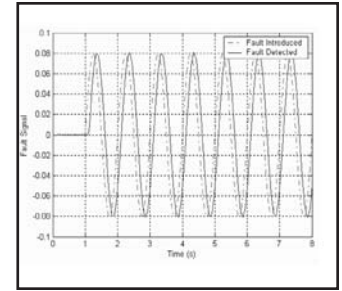


Figure 12. Fault ID in Sensor 4

from 5 Hz to 20 Hz for 100 seconds. Using the MATLAB System Identification toolbox, we obtain a state space model of second order. The validation results of the identified models from the shaker to sensors are with the fit of 88.44%, 87.83%, 87.49% and 87.16%, respectively. Similarly, the validation results of the identified models from the controllers to sensors are with the fit of 88.09%, 89.45%, 90.55% and 92.21%, respectively.

## Results

Some research on the FTC turns to the design of an integrated filter and controller. Such methods usually lead to very high order integrated filter/controllers. To ease the implementation of the FTC, we designed the  $H_\infty$  FDI filter and  $H_\infty$  FTC separately.

In the literature regarding  $H_\infty$  norm analysis and design, the weight function selection is a highly problem-dependent, iterative process. Usually, we repeat the trial-and-error procedure until the desired performance and robustness objectives are achieved. In this research, the control objective is to suppress the first mode vibration of the smart structure. The disturbance input weight,  $W_d$ , is used to shape the worst case disturbance. It can be a low pass shape, or it can stress the amplitude at the frequency of the first mode. In this research, we use the  $a$  sine wave with the frequency of the first mode of the structure as the disturbance input. So the disturbance input weight is set to scalar 1. The performance or error weight,  $W_e$ , is used to specify the performance objectives of the resulting filter from the  $H_\infty$  optimization, *e.g.*, bandwidth, steady-state requirements, and attenuation/amplification of signals at certain frequency ranges. Typically, the actuator fault input weight,  $W_f$  has low-pass characteristics to emphasize the

effects of faults on the low-frequency domain.

The identified structure model combined with the weights yields the augmented model,  $P$ , as shown in the black square in Fig. 1. Using the developed LMI method, the following four stable, 3rd order  $H_\infty$  filters for sensors are obtained. Using the a similar technique, the reduced 3rd  $H_\infty$  controller is obtained.

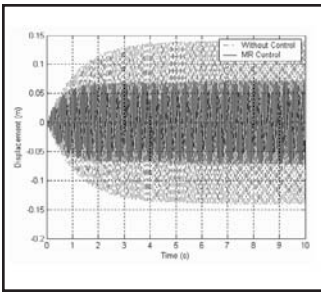
The truss was continuously excited by the sine wave at 85.95 rad/sec, which is the worst case disturbance input. Residual responses to different faulty inputs were examined. First, a faulty signal of square wave with different varying amplitude was introduced into four sensors at 10 sec. Filter outputs are shown in Fig. 5 to Fig. 8. From these figures, we can see that the residual signal clearly identified the square wave.

For comparison, another simulation with the sine wave faulty input was conducted. Fig. 9 to Fig. 12 shows the faulty input signal and the residual signal. It can be seen that within one second of the occurrence of the fault, the residual signal clearly indicated the fault.

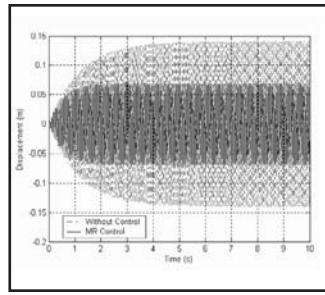
Next, the designed  $H_\infty$  FTC controller was examined using the MR damper. The structure was excited by a sine wave at 85.95 rad/sec. To simulate the partial failure of sensors, the outputs of the sensor 1, 2, 3 and 4 were reduced to 10%, 20%, 30%, and 40%, respectively, after 5 seconds had elapsed.

The semi-active control law of the MR damper is:

$$u_d = \begin{cases} c_d \dot{x} + f_{dy\max} \operatorname{sgn}(\dot{x}) & (u\dot{x} < 0 \text{ and } |u| > u_{d\max}) \\ |u| \operatorname{sgn}(\dot{x}) & (u\dot{x} < 0 \text{ and } |u| < u_{d\max}) \\ c_d \dot{x} + f_{dy\min} \operatorname{sgn}(\dot{x}) & u\dot{x} \geq 0 \end{cases}$$



**Figure 13. Displacement at Tip of Truss (With Faults)**



**Figure 14. Displacement at Tip of Truss (Without Faults)**

where  $u_d$  is the control force applied by the MR damper and  $u$  is the optimal force acquired by FTC. The parameters of the MR damper are:

$$c_d = 2.8332 \times 10^3 \text{ N} \cdot \text{s/m}$$

and

$$c_d = 2.8332 \times 10^3 \text{ N} \cdot \text{s/m} .$$

The displacement at the tip of the truss is shown in Fig. 13. It can be seen that the response of the closed-loop system with the FTC controller is reduced by about 50 percent, though there are partial faulty signals in the output of the sensors. The result of the FTC Controller without faults in sensors is shown in Fig. 14. Comparing the two figures, it can be clearly seen that the system with FTC controller remained almost unchanged when a failure occurred in the sensors.

## Conclusions

In this research, a model-based fault detection and isolation (FDI) and the solution of the fault tolerant control problem using  $H_\infty$  techniques are considered for the vibration control of truss structures. A linear matrix inequality (LMI) formulation is obtained to design a full order robust  $H_\infty$  filter to estimate the faulty input signals. FDI LMI synthesis conditions are obtained by applying the Bounded Real Lemma to the closed loop system. A feasible solution for these conditions forms a convex problem for the full order filter, which is solved via LMI optimization techniques. A fault tolerant  $H_\infty$  controller, which minimizes the control objective selected in the presence of disturbances and faults, has been designed for the combined system of plant and filter. An 8-bay truss structure has been used in validation of the FDI filter and FTC controller designs. A MR damper was installed in the truss to reduce the vibration of the truss using the FTC controller. To assist control system design, system identification was conducted for the first vibration mode of the truss. The state space model from system identification was used for the  $H_\infty$  filter design. Residuals obtained from the filter through simulation are seen following the faults signals. The result shows that the designed FTC controller can achieve a fair vibration reduction ratio, though there are partial faulty signals in the output of sensors.

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