

A Theoretical Analysis of Vibrational Modes Aimed at their use as Measures of Bone Damage

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SCATTERING IS ONE OF THE MOST WIDELY used techniques for probing the interior of solids. In order to adapt this methodology to extract material properties of cortical and trabecular bone tissue, we introduced a model system consisting of a porous segment constructed from a digitized image of vertebral trabecular bone and a solid segment attached to it on a side. In our calculations, the rear end will be held stationary while the solid front end will be subjected to a short-time pulse of a known shape. The stress on the front end following the pulse will be calculated using an integration of the type adopted for laborator procedures. Figure 1 shows an example of the calculated response for such a model system when the externally applied pulse is triangular. The question then is whether this time-dependent response can be inverted to obtain “effective” material properties of cortical and trabecular segments.

We approximated the system with a one-dimensional array of springs-and-masses. The trabecular bone is represented by N_1 identical objects of mass m_1 connected by elastic elements of spring constant k_1 , and are assumed to move in a medium with damping η_1 . Cortical bone is represented by η_2 identical objects of mass m_2 connected by elastic elements of spring constant k_2 , and are assumed to move in a medium with damping η_2 . The scattering problem assumes that the pulse sent inward is partially reflected back to the cortical bone and partially transmitted to the trabecular bone. Solving the problem, one finds that the Fourier transform of the stress on the front end is given by

$$\chi = x_0(\omega)(w^{-\mu_2} + re^{\mu_2})/(1+r),$$

with the reflection coefficient

$$r = \exp(-2\mu_2 N_2)(\gamma \sinh \mu_2 - \sinh \mu_1)/(\gamma \sinh \mu_2 + \sinh \mu_1).$$

Here $\gamma = k_2/k_1$

and for $j=1$ or 2 , $\mu = \cosh^{-1}(1 - \omega^2/2c_j^2 + i\omega\gamma_j/2c_j^2)$,

where $\gamma_j = \eta_j/m_j$, and $c_j^2 = k_j/m_j \cdot x_o(\omega)$

is the Fourier transform of the pulse. We propose to develop inverse techniques to estimate c_1 , c_2 , γ_1 , γ_2 , k_1 and k_2 from the time behavior of c .



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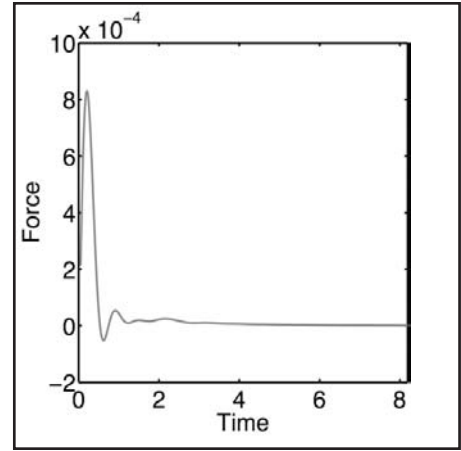


Figure 1. The Linear Response to a Triangular Pulse

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- Espinoza Ortiz, J. S., C. S. Rajapakse, and G. H. Gunaratne. “Strength Reduction in Electrical and Elastic Networks,” *Phys. Rev. B*, 66 (2002): 144203.
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- Song, Y. and G. H. Gunaratne. “A Method for Vibrational Assessment of Solid Bone,” *Chaos* 16 (2006): 033102.

Presentations

- “How to Find When Bones May Break” Workshop on Biomechanics, Notre Dame University, November 2002; Georgia Institute of Technology, March 2003; Boston University, February 2004; University of Connecticut, April 2004.
- “Summer School for Advanced Electronic and Bio-materials,” International Center for Materials Physics, Chinese Academy of Sciences, Shenyang, China, June 2004; Dalian University, China, June 2004.

Proposals

- “Development of Analytical Tools for Vibrational Assessment of Bone,” Joint NSF/NIH program for Mathematical Biology, \$1,467,000: NSF ranked the proposal in the “Highest Priority”, but it was not funded by the NIH. (*The program officer requested that it be resubmitted in June 2006.*)
- “Vibrational Response as a Measure of Trabecular bone Quality,” NIH, \$1,471,000. (*Not funded.*)
- “Theoretical Analysis of Acoustical Assessment of Bone,” NSF, \$220,000. (*Not funded; to be resubmitted.*)