Energy-to-Peak Induced Norm Upper Bound Control Approach for Collocated Structural Systems

by Mona Meisami-Azad, Javad Mohammadpour, and Karolos M. Grigoriadis

ABSTRACT—UH researchers determine an analytical upper bound for the energy-to-peak gain or L_2–L_∞ induced norm of collocated structural systems. The proposed technique does not require solving Lyapunov equations or a set of LMIs usually needed to compute the L_2–L_∞ induced norm. Furthermore, an output feedback control law has been developed which ensures that the closed loop system of the collocated structure and the output feedback control achieves a desired L_2–L_∞ norm bound.

Karolos M. Grigoriadis

A USEFUL MEASURE OF PERFORMANCE OF A DYNAMICAL system is the induced L_2–L_∞ or energy-to-peak norm that dictates the peak value (L_∞norm) of the system subject to a bound on the energy (L_2 norm) of the input.[1] Computation of the L_2–L_∞ norm of a system using the linear matrix inequality (LMI) formulation or the Lyapunov equation approach can be very intensive, especially for large scale systems. The LMI problem has a polynomial time complexity with respect to the number of decision variables, while solution of Lyapunov equations is of quadratic complexity with respect to computations and storage requirements. Consequently, the use of these tools for performance analysis and control of large scale systems is limiting.[2]

The present work examines the induced L_2–L_∞ analysis problem and the corresponding output feedback control design for collocated structural systems. Toward this purpose, we consider the LMI analysis conditions that characterize the L_2–L_∞ norm of the system and propose a particular solution for the Lyapunov function in the LMIs. The proposed bound is shown to be exact for the one-degree-of-freedom structure. This work demonstrates the high accuracy of the proposed upper bound on the induced L_2–L_∞ norm of the system that can be calculated effectively. The qualifications of the proposed upper bound and the control design methodology are verified using a large scale model of the International Space Station (ISS).

Consider a structural system with collocated sensors and actuators in a second order vector form represented by

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = F(u(t) + w(t)) \quad (1)$$
$$y(t) = F^T\dot{q}(t)\,,$$

where

$$F \in \mathbb{R}^{n\times r},\ M = M^T > 0,\ K = K^T > 0,\ D = D^T > 0$$

represent input disturbance matrix with full column rank, mass matrix, stiffness matrix, and damping matrix, respectively. The vectors

$$q(t) \in \mathbb{R}^n,\ \ u(t) \in \mathbb{R}^r,\ \ w(t) \in \mathbb{R}^r,\ \text{and}\ \ y(t) \in \mathbb{R}^r$$

are the vector of displacements, control input, external/disturbance signal, and measured output, respectively.

For a fixed initial condition $x(0) = 0$, the induced energy-to-peak gain is defined as

$$\|T_{yu}\|_{L_2-L_\infty} = \sup_{u\in l_2} \frac{\|y\|_\infty}{\|u\|_2}\,. \quad (2)$$

The following result provides an explicit formula for computation of the induced L_2–L_∞ norm upper bound of collocated structural systems that only needs the calculation of the maximum eigenvalue of matrices generated from the system data.

Theorem 1: Consider the system in (1). This system has an induced L_2–L_∞ norm ν from the input $u(t)$ to the output $y(t)$ that satisfies the following bound

$$\nu \le \nu_{bound} = \frac{1}{\sqrt{2}}[\lambda_{max}(F^T D^{-1} F)\lambda_{max}(F^T M^{-1} F)]^{1/2}\,. \quad (3)$$

It is worth mentioning that for the single degree-of-freedomcase (n=1), the proposed upper bound for the induced L_2–L_∞ performance is exactly the same as the actual induced L_2–L_∞ norm of the system.[3]

Next, consider the collocated system in (1) along with a controlled output equation

$$z(t) = F^T\dot{q}(t)\,. \quad (4)$$

The collocated L_2–L_∞ control synthesis problem is to design a symmetric static output feedback gain $H = H^T$ such that the output feedback control law

$$u(t) = -Hy(t) \quad (5)$$

renders the closed-loop system stable with an L_2–L_∞ norm less than a prescribed scalar $\nu > 0$, *i.e.*

$$\|T_{zw}\|_{L_2-L_\infty} \le \nu\,, \quad (6)$$

where T_{zw} is the closed-loop transfer function mapping $w(t)$ to $z(t)$.

We now present the following result that provides an explicit expression for the output feedback gain H that guarantees a closed-loop L_2–L_∞ norm less than a bound ν.

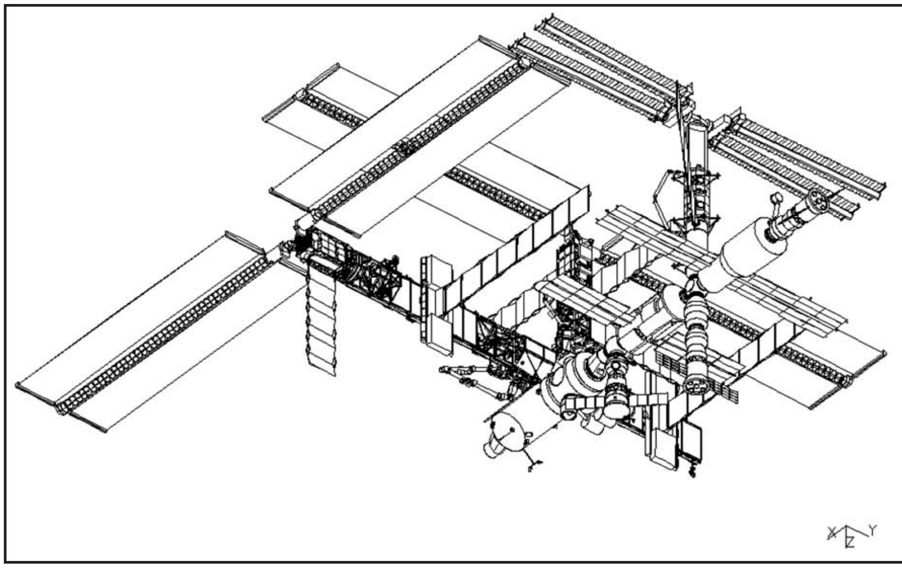

Figure 1: Assembly Phase 8A-OBS of the ISS Model

Theorem 2: A control law as in (5) whose interconnection with the collocated system (1) satisfies the performance bound (6) is derived as following: a) If F is square and invertible, H can be computed as

$$H \geq \eta I - F^{-1} D F^{-T}\,, \qquad (7)$$

where

$$\eta = \frac{1}{2\nu^2} \lambda_{max}(F^T M^{-1} F)\,. \qquad (8)$$

(b) If FFT is singular, H can be computed as

$$H \geq \eta I + (F^{\dagger} D F^{\perp^T})(F^{\perp} D F^{\perp^T})^{-1}(F^{\perp} D F^{\dagger^T}) - F^{\dagger} D F^{\dagger^T}\,, \qquad (9)$$

where η is given by (8).

Results

We apply the proposed L_2–L_∞ bound analysis and output controller design to a very large scale structural system model. We consider the finite element structural model for the assembly phase 8A-OBS of the International Space Station (ISS) with collocated control, Rayleigh damping, and 720 states, as shown in Fig. 1. Computation of the induced L_2–L_∞ (energy-to-peak) norm through solving the Lyapunov equation requires 16.548 seconds; the obtained norm is equal to 5.9051. However, it only takes 0.3607 seconds to calculate the norm bound via the proposed analytical bound approach which provides a bound equal to 5.935.We observed that the bound estimates the norm closely and its calculation is computationally efficient. Table 1 shows the results of the computations performed in order to determine the energy-to-peak feedback control gains and the required computational times for different desired closed-loop norm bounds. For instance, designing an output feedback controller to guarantee that the energy-to-peak norm of the closed-loop system is less than or equal to 0.5 takes 0.2642 seconds. The actual norm of the closed-loop system with the designed controller has been determined to be 0.4998. It takes 9.1099 seconds to calculate this norm using MATLAB. This is almost 65 times longer than our proposed method to determine the energy-to-peak norm bound computed in 0.1349 seconds. Notice that the computation of an L_2–L_∞ controller for this system using standard methods fails since a solution by Lyapunov equations or LMIs for a system of this size is computationally prohibitive.

References

[1]C. Scherer and S. Weiland. "Lecture Notes DISC Course on Linear Matrix Inequalities in Control," Version 2.0, April 1999.

[2]R.E. Skelton, T. Iwasaki, and K. M. Grigoriadis. *A Unified Algebraic Approach to Linear Control Design*. London; Bristol, PA: Taylor & Francis, 1998.

[3]M. MeisamiAzad, J. Mohammadpour, and K.M. Grigoriadis. "Energy-to-Peak Norm Upper Bound for Collocated Structural Systems," scheduled for publication in *Proc.* SPIE 14th Annual International Symposium on Smart Structures and Materials, San Diego, CA, March 2007.

Table 1: Results and Comparisons for the 8A-OBS ISS Model

Desired closed-loop L_2–L_∞ norm bound	**Time to calculate feedback gain using Th. 2 (sec)**	**Exact L_2–L_∞ norm of the closed-loop system**	**Time to calculate exact L_2–L_∞ norm (sec)**	**Time to calculate L_2–L_∞ norm bound (sec)**
5	0.2650	4.9910	9.1441	0.1294
1	0.2645	0.9993	9.1358	0.1406
0.5	0.2642	0.4998	9.1099	0.1349
0.1	0.2642	0.1	9.1034	0.1319

TRUSS—Dr. Heidar Malki (*l.*) and Dr. Karolos M. Grigoriadis have collaborated in past ISSO projects, including "A Neural-Network-Based Approach for Control of Vibration in a Black Hawk Helicopter." Here, they stand with a mechanical truss used to simulate vibrations and measure their effects.

RESEARCH HUB—Hoffman Hall is a major UH research center housing the Dept. of Computer Science, the Dept. of Mathematics, and the innovative Texas Learning and Computation Center (TLC²).

Publications

Bai, Y., and K.M. Grigoriadis. "H_∞ Collocated Control of Structural Systems: An Analytical Bound Approach," AIAA *Journal of Guidance, Control and Dynamics* 28.5 (2005): 850853.

Hiramoto, K., and K.M. Grigoriadis. "Integrated Design of Structural and Control Systems with a Homotopylike Iterative Method," in *Proc.* American Control Conference, (2005).

Hiramoto, K., Y. Bai, and K.M. Grigoriadis. "Upper Bound H_∞ and H_2 Control for Symmetric Mechanical Systems," *Proc*. International Federation of Automatic Control World Congress, (2005).

Hiramoto, K., and K.M. Grigoriadis. "Integrated Design of Structural and Control Systems with a Homotopylike Iterative Method," *International Journal of Control* 79.9 (2006): 1062-73.

MeisamiAzad, M., J. Mohammadpour, and K. M. Grigoriadis. "An H_2 Upper Bound Approach for Control of Collocated Structural Systems," *Proc*. American Control Conference, New York, NY, June 2007. (*Accepted*.)

MeisamiAzad, M., J. Mohammadpour, and K. M. Grigoriadis. "Energy-to-Peak Norm Upper Bound for Collocated Structural Systems," *Proc*. SPIE 14th Annual International Symposium on Smart Structures and Materials, San Diego, CA, March 2007. (*Accepted.*)

Mohammadpour, J., M. Meisami-Azad, and K. M. Grigoriadis. "An Efficient Approach for Damping Parameter Design in Collocated Structural Systems Using An H_2 Upper Bound," *Proc*. American Control Conference, New York, NY, June 2007. (*Accepted*.)

Mohammadpour, J., M. Meisami-Azad, and K. M. Grigoriadis. "An Efficient Approach for Integrated Design of Damping and Feedback Controllers: H_2 and H_∞ Perspectives," *Structural and Multidisciplinary Optimization*, Feb. 2007. (*To be submitted*.)

Presentations

Hiramoto, K., and K.M. Grigoriadis. "Integrated Design of Structural and Control Systems with a Homotopy Like Iterative Method," in *Proc*. American Control Conference, Portland, OR, 2005.

Mehendale, C. and K. M. Grigoriadis. "Control of Timedelayed LPV Systems Using Delayed Feedback," in *Proc*. International Federation of Automatic Control World Congress, Prague, Czech Republic, 2005.